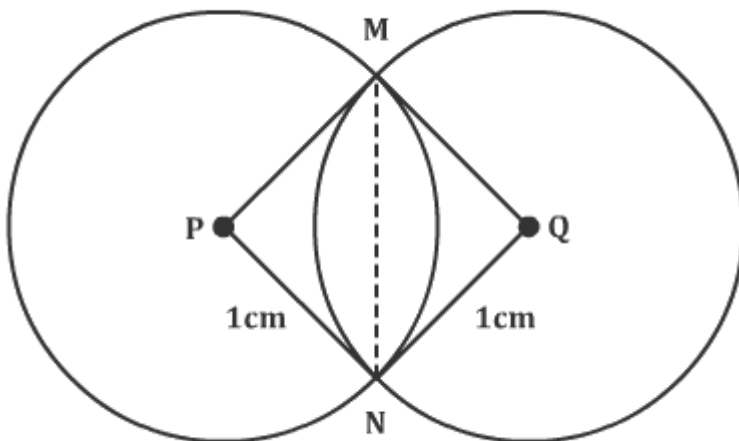


Solutions

1. $x = 16^3 + 17^3 + 18^3 + 19^3$
 $= (16^3 + 19^3) + (17^3 + 18^3)$
 $= (16 + 19)(16^2 + 16 \times 19 + 19^2) + (17 + 18)(17^2 + 17 \times 18 + 18^2)$
 $= 35 \times (\text{an odd number}) + 35 \times (\text{another odd number}) = 35 \times (\text{an even number})$
 $= 35 \times (2k) \dots (k \text{ is a positive integer})$
 $\therefore x = 70k$
 $\therefore x$ is divisible by 70.
 Remainder when x is divided by 70 = 0
 Hence, option 1.

2. The change in the amount of chemical in each tank after every minute is as follows:
 A: $-20 - 10 + 90 = 60$
 B: $-100 + 110 + 20 = 30$
 C: $-50 - 90 + 100 = -40$
 D: $-110 + 10 + 50 = -50$
 Since tank D loses the maximum amount of chemical in a minute, it will be emptied first.
 Let n minutes be the time taken by tank D to get empty.
 $\therefore 1000 - 50n = 0$
 $\therefore n = 20$ minutes
 Hence, option 3.

3.



Let the two circles with centres P and Q intersect at M and N.
 Quadrilateral PQMN is a square.
 $m\angle MPN = m\angle MQN = 90^\circ$
 The area common to both the circles = $2(\text{Area of sector P-MN} - \text{Area of } \Delta PMN)$

$$= 2[(90/360 \times \pi \times 1^2) - (1/2 \times 1^2)]$$

$$= \pi/2 - 1$$

Hence, option 2.

4. Let r be the radius of the circular tracks.

Length and breadth of the rectangular track are $4r$ and $2r$ respectively.

Length (perimeter) of the rectangular track = $12r$

Length of the two circular tracks (figure of eight) = $4\pi r$

If A and B have to reach their starting points at the same time,

$$\frac{12r}{a} = \frac{4\pi r}{b}$$

(where a and b are the speeds of A and B respectively)

$$\therefore \frac{b}{a} = \frac{4\pi}{12}$$

$$\therefore (b - a) \times 100/a = 0.047 \times 100$$

$$= 4.7\%$$

Hence, option 4.

5. Let there be g girls and b boys.

Number of games between two girls = ${}^g C_2$

Number of games between two boys = ${}^b C_2$

$$\therefore g(g - 1)/2 = 45$$

$$\therefore g^2 - g - 90 = 0$$

$$\therefore (g - 10)(g + 9) = 0$$

$$\therefore g = 10$$

Also,

$$b(b - 1)/2 = 190$$

$$\therefore b^2 - b - 380 = 0$$

$$\therefore (b + 19)(b - 20) = 0$$

$$\therefore b = 20$$

$$\therefore \text{Total number of games} = (g + b)C_2 = {}^{30}C_2 = 435$$

$$\therefore \text{Number of games in which one player is a boy and the other is a girl} = 435 - 45 - 190 = 200$$

Hence, option 1.

6. Ram and Shyam run a race between points A and B, 5 km apart. Ram starts at 9 a.m. from A at a speed of 5 km/hr, reaches B, and returns to A at the same speed. Shyam starts at 9:45 a.m. from A at a speed of 10 km/hr, reaches B and comes back to A at the same speed.

Ram starts at 9:00 am and Shyam starts at 9:45 am from A.

Ram reaches B at 10:00 am (as his speed is 5km/hr and the distance between A and B is 5km)

When Ram reaches B, Shyam is $15/60 \times 10 = 2.5$ km away from A.

Ram meets Shyam $(2.5 \times 60)/(10 + 5)$ minutes after 10:00 a.m. i.e., at 10:10 a.m.

Shyam reaches B at 10:15 a.m.

At 10:15 a.m., Ram is $(15/60) \times 5 = 1.25$ km away from B.

Shyam overtakes Ram in $1.25/(10 - 5) = 0.25$ hrs = 15 minutes after 10:15 am i.e. at 10:30 a.m.

Hence, option 2.

7. Shyam overtakes Ram at 10:30 a.m.

Hence, option 2.

8.

$$R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$$

$$\therefore a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$$

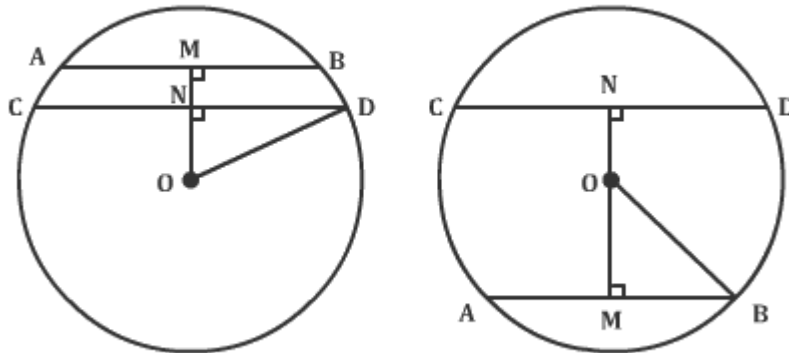
$$\therefore R = \frac{(30 - 29)[30^{64} + (30^{63} \times 29) + \dots + 29^{64}]}{30^{64} + 29^{64}}$$

$$\therefore 30^{64} + 30^{63} \times 29 + \dots + 29^{64} > 30^{64} + 29^{64}$$

$$\therefore R > 1$$

Hence, option 4.

9.



The two chords AB and CD can be on the same side or the opposite sides of the centre O.

Let M and N be the midpoints of AB and CD.

\therefore MN is the distance between the two chords.

MB = 12 cm and ND = 16 cm

OM and ON are perpendicular to AB and CD respectively.

$\therefore ON^2 = 20^2 - 16^2$ (By Pythagoras theorem)

$\therefore ON = 12$ cm

Similarly, OM = 16 cm

Case 1: AB and BC are on the same side of the centre.

$$MN = OM - ON = 4 \text{ cm}$$

Case 2: $MN = OM + ON = 28 \text{ cm}$

Hence, option 4.

10. We have, $x^2 = y^2$ and $(x - k)^2 + y^2 = 1$

Solving the two equations simultaneously, we get,

$$(x - k)^2 + x^2 = 1$$

$$\therefore x^2 - 2kx + k^2 + x^2 = 1$$

$$\therefore 2x^2 - 2kx + (k^2 - 1) = 0$$

If this equation has a unique solution for x , then discriminant = 0

$$\therefore 4k^2 - 8(k^2 - 1) = 0$$

$$\therefore 8 - 4k^2 = 0$$

$$\therefore k^2 = 2$$

$$\therefore k = \pm\sqrt{2}$$

Since k is positive the other solution is ruled out

$$\therefore k = \sqrt{2}$$

Hence, option 3.

11. $p = (1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!) + \dots + (10 \times 10!)$

$$\text{Now, } n \times n! = [(n + 1) - 1] \times n! = (n + 1)! - n!$$

$$\therefore p = 2! - 1! + 3! - 2! + 4! - 3! + 5! - 4! + \dots + 11! - 10!$$

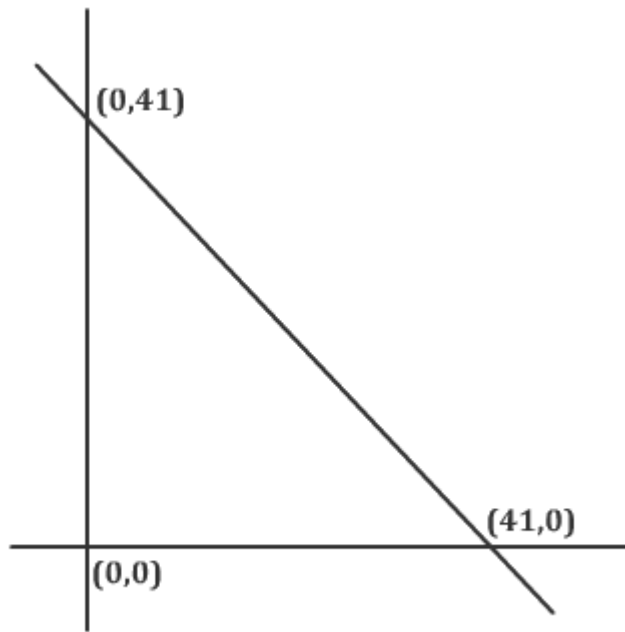
$$\therefore p = 11! - 1! = 11! - 1$$

$$\therefore p + 2 = 11! + 1$$

$\therefore p + 2$ when divided by $11!$ leaves a remainder of 1.

Hence, option 4.

12.



The points satisfying the equations $x + y < 41$, $y > 0$, $x > 0$ lie inside the triangle.

Integer solutions of $x + y < 41$ can be found as follows.

If $x + y = 40$

(x, y) (1, 39), (2, 38), ..., (39, 1) ... (39 solutions)

If $x + y = 39$

(1, 38), (2, 37), ..., (38, 1) ... (38 solutions)

If $x + y = 38$, we get 37 solutions and so on till $x + y = 2$... (1 solution)

Thus there are $39 \times 40 / 2 = 780$ integer solutions to $x + y < 41$

The number of points with integer coordinates lying inside the circle = 780

Hence, option 1.

13. Let $A = 100x + 10y + z$ ($x \neq 0$, x, y, z are single digit numbers)

$$\therefore B = 100z + 10y + x$$

$$\therefore B - A = 99(z - x)$$

As $(B - A)$ is divisible by 7 and 99 is not, $(z - x)$ is divisible by 7

$\therefore z$ and x can have values (8, 1) or (9, 2)

y can have any value from 0 to 9.

$$A = 1y8 \text{ or } 2y9$$

\therefore Lowest possible value of A is 108 and the highest possible value of A is 299.

Hence, option 2.

14. $a_1 = 1$

$$a_{n+1} = 4n + 3a_n - 2$$

$$a_2 = 4 - 2 + 3(1) = 5 = 3^2 - 1$$

$$a_3 = 4(2) + 3(5) - 2 = 21 = 3^3 - 6$$

$$a_4 = 4(3) + 3(21) - 2 = 73 = 3^4 - 8$$

$$\therefore a_n = 3^n - 2(n)$$

$$\therefore a_{100} = 3^{100} - 200$$

Hence, option 3.

15. Let O and E represent odd and even digits respectively.

\therefore S can have digits of the form
 O _ O _ E or O _ E _ O or E _ O _ O

Case 1: O _ O _ E

The first digit can be chosen in 3 ways out of 1, 3 and 5

The third can be chosen in 2 ways.

The fifth digit can be chosen in 2 ways after which the second and fourth digits can be chosen in 2 ways.

\therefore There are $3 \times 2 \times 2 \times 3 = 24$ ways in which this number can be written. 12 out of these ways will have 2 in the rightmost position and 12 will have 4 in the rightmost position.

\therefore The sum of the rightmost digits in Case 1 = $(12 \times 2) + (12 \times 4) = 72$

Case 2: O _ E _ O

This number can again be written in 24 ways.

8 out of these ways will have 1 in the rightmost position, 8 will have 3 in the rightmost position and 8 will have 5 in the rightmost position.

Thus the sum of the rightmost digits in Case 2 = $(8 \times 1) + (8 \times 3) + (8 \times 5) = 72$

Case 3: E _ O _ O

This number can also be written in 24 ways.

As in Case 2, 8 out of these ways will have 1 in the rightmost position, 8 will have 3 in the rightmost position and 8 will have 5 in the rightmost position.

\therefore The sum of the rightmost digits in Case 3 = $(8 \times 1) + (8 \times 3) + (8 \times 5) = 72$

\therefore The sum of the digits in the rightmost position of the numbers in S = $72 + 72 + 72 = 216$

Hence, option 2.

16. $30^{2720} = 3^{2720} \times 10^{2720}$

The rightmost non zero digit of 30^{2720} will be the digit in the unit's place of 3^{2720} .

3's power cycle is 3, 9, 7, 1 and cyclicity is 4.

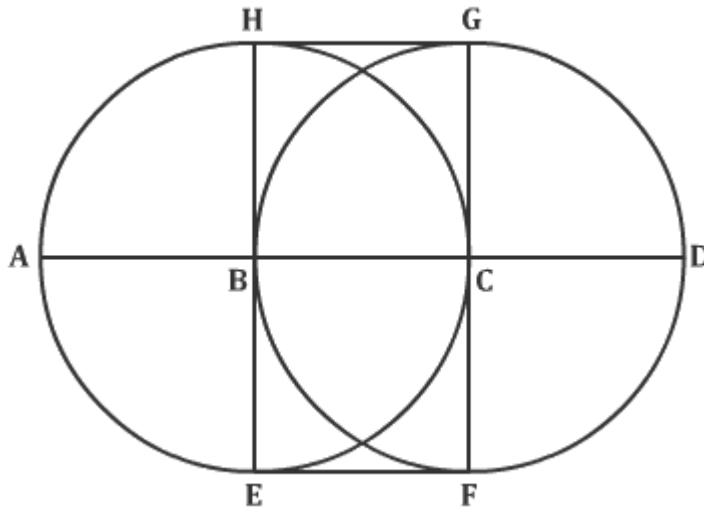
$$2720 = 680 \times 4$$

\therefore The digit in the unit's place of 3^{2720} is 1.

\therefore The rightmost non-zero digit of 30^{2720} is 1.

Hence, option 1.

17.



The ant will not go into the circles with centers B and C and radius = 1 m
 The minimum distance that the ant has to traverse = the distance of the path A-H-G-D

$$HG = 1\text{ m}$$

$$AH = GD = \frac{1}{4} \times \text{Circumference of Circle} = \frac{\pi}{2}$$

$$AH + GD = \pi \text{ m}$$

\therefore The ant must traverse $1 + \pi \text{ m}$

Hence, option 2.

18.

$$\begin{aligned} \log_x \left(\frac{x}{y} \right) + \log_y \left(\frac{y}{x} \right) &= \frac{\log x - \log y}{\log x} + \frac{\log y - \log x}{\log y} \\ &= 1 - \log_x y + 1 - \log_y x \\ &= 2 - \log_x y - \log_y x \\ &= 2 - (\log_x y + \log_y x) \end{aligned}$$

As $x \geq y$ and $y > 1$,

$$\log_y x \geq 1 \text{ and } \log_x y \leq 1$$

$$\log_y x + \log_x y > 1$$

$$\therefore 2 - (\log_x y + \log_y x) < 1$$

$$\therefore \log_x \left(\frac{x}{y} \right) + \log_y \left(\frac{y}{x} \right) \neq 1$$

Hence, option 4.

19. n can be a 2 digit or a 3 digit number.

Case (I)

Let n be a 2 digit number.

Let $n = 10x + y$, where x and y are non-negative integers,

$$P_n = xy \text{ and } S_n = x + y$$

Now, $P_n + S_n = n$

$$\therefore xy + x + y = 10x + y$$

$$\therefore xy = 9xy = 9$$

There are 9 two digit numbers (19, 29, 39, ..., 99) for which $y = 9$

Case (II)

Let n be a 3 digit number.

Let $n = 100x + 10y + z$, where x, y and z are non-negative integers,

$$P_n = xyz \text{ and } S_n = x + y + z$$

Now, $P_n + S_n = n$

$$xyz + x + y + z = 100x + 10y + z$$

$$\therefore xyz = 99x + 9y$$

$$\therefore z = 99/y + 9/x$$

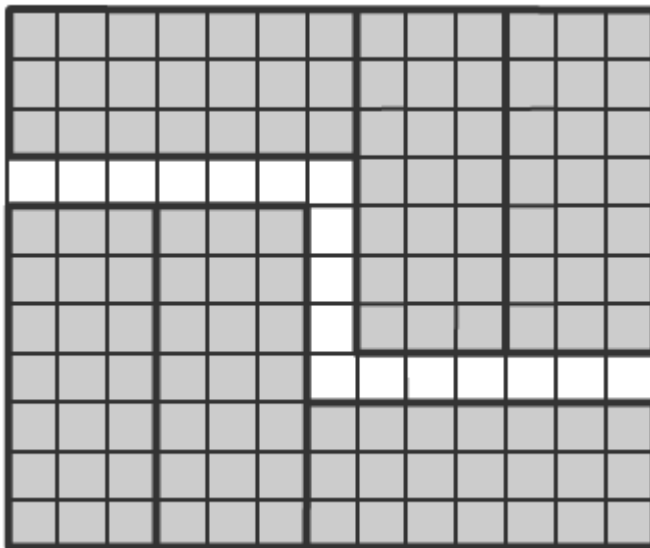
From the above expression, $0 < x, y < 9$

But, we cannot find any value of x and y , for which z is a single digit number.

\therefore There are no 3 digit numbers which satisfy $P_n + S_n = n$

Hence, option 4.

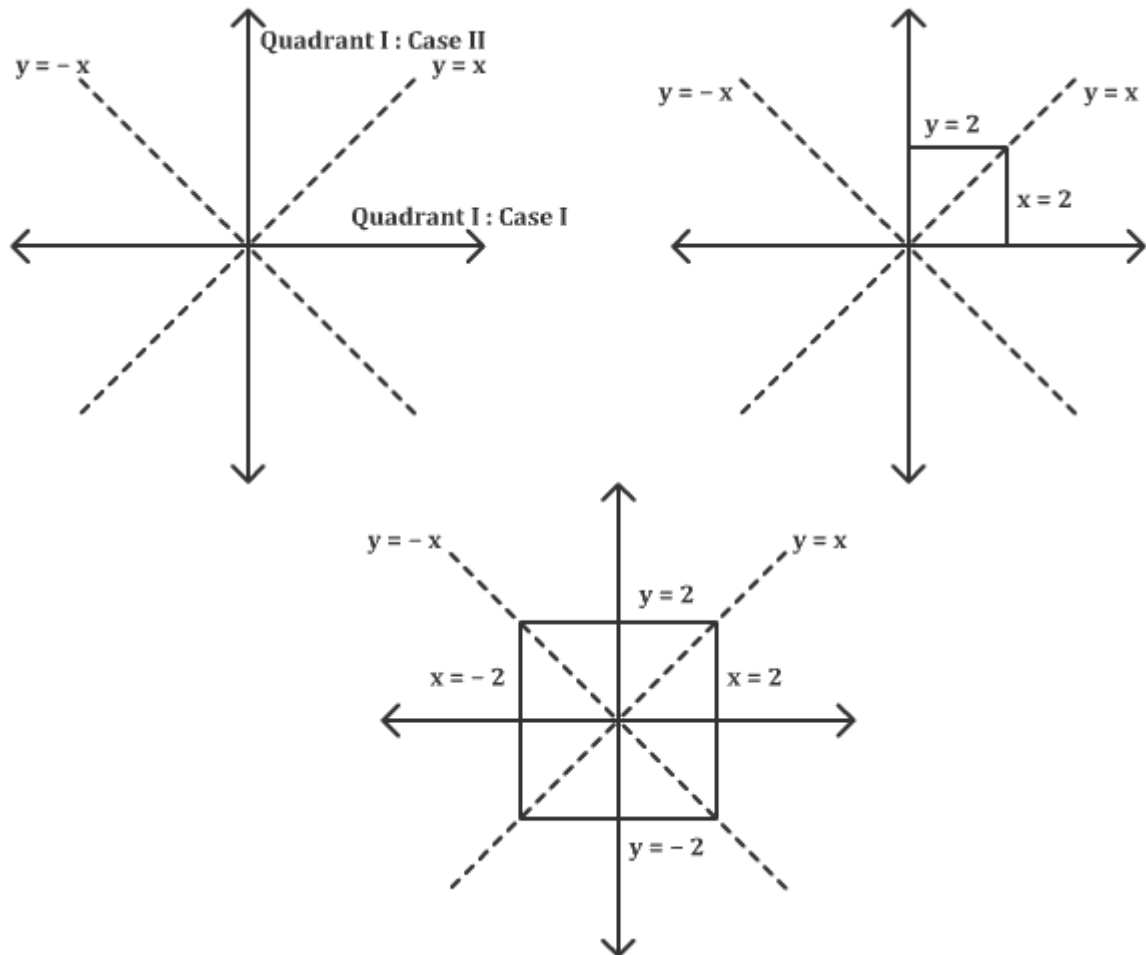
20.



This problem can be solved by trying different ways of placing the tiles on the floor. The maximum number of tiles that can be accommodated is 6 as shown in the figure.

Hence, option 3.

21.



$$|x + y| + |x - y| = 4 \quad \dots (i)$$

Consider the case when x and y are both positive. This is the area of quadrant I
In this case two cases are possible.

Case I: $x > y$

In this case expression (i) becomes

$$x + y + x - y = 4$$

$$\therefore 2x = 4$$

$$\therefore x = 2$$

Case II: $x < y$

In this case expression (i) becomes

$$x + y + y - x = 4$$

$$\therefore 2y = 4$$

$$\therefore y = 2$$

\therefore The area for the first quadrant is as shown in the figure.

Extending the same logic to other quadrants we get the area as shown in the diagram.

$$\therefore \text{Its area} = 4 \times 4 = 16 \text{ sq. units}$$

Hence, option 3.

22. $AO = OD = 1.5 \text{ cm}$

$$AE + EB = 3 \text{ cm and } AE:EB = 1:2$$

$$\therefore AE = 1 \text{ cm and } EB = 2 \text{ cm}$$

$$OE = AO - AE = 1.5 - 1 = 0.5 \text{ cm}$$

$$\text{Similarly, } NL = 1 \text{ cm, } M = 2 \text{ cm and } OL = 0.5 \text{ cm}$$

OEHL is a square as all its angles are right angles and $OE = OL$

$$\therefore EH = HL = 0.5 \text{ cm}$$

$$\text{In } \triangle ODL, OD^2 = OL^2 + DL^2$$

$$1.5^2 = 0.5^2 + (0.5 + DH)^2$$

$$2.25 = 0.25 + 0.25 + DH + DH^2$$

$$DH^2 + DH - 1.75 = 0$$

$$DH = \frac{-1 \pm \sqrt{1 - 4(-1.75)}}{2}$$

$$= \frac{(2\sqrt{2} - 1)}{2} \quad (DH > 0)$$

Hence, option 2.

23. $m\angle BCD = m\angle BAC$ and B is common to triangles ABC and CBD.

$\triangle ABC$ is similar to $\triangle CBD$

$$AB/CB = BC/BD = AC/CD$$

$$AB/12 = 12/9 = AC/6$$

$$AB = 16 \text{ cm and } AC = 8 \text{ cm}$$

$$AD = AB - BD = 16 - 9 = 7 \text{ cm}$$

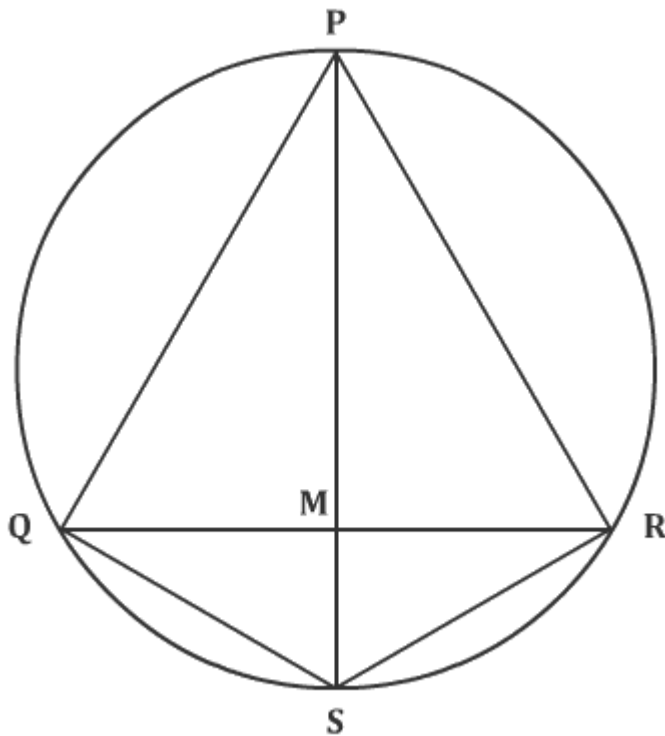
$$\therefore \text{Perimeter of } \triangle ABC = 7 + 6 + 8 = 21 \text{ cm}$$

$$\therefore \text{Perimeter of } \triangle BDC = 9 + 6 + 12 = 27 \text{ cm}$$

$$\therefore \text{Required ratio} = 21/27 = 7/9$$

Hence, option 1.

24.



ΔPQR is an equilateral triangle and PS is the diameter.
 $\therefore m\angle PQS = m\angle PRS = 90^\circ$ (angles subtended in a semi-circle)
 PS bisects QR as it is the median of ΔPQR
 $m\angle PMQ = m\angle PMR = 90^\circ$
 $\therefore m\angle QPS = m\angle RPS = 30^\circ$
 $\therefore m\angle PSQ = m\angle PSR = 60^\circ$

Radius = r
 $\therefore PS = 2r$
 As $\Delta PQS, \Delta PQM, \Delta MQS$ are $30^\circ-60^\circ-90^\circ$ triangles,
 $QS = r, PQ = \sqrt{3}r$
 Similarly, $RS = r, PR = \sqrt{3}r$
 \therefore Perimeter of quadrilateral $PQRS = 2r + 2\sqrt{3}r = 2r(1 + \sqrt{3})$
 Hence, option 1.

25. n will be of the form $11ab$, where a and b are odd numbers.

We are looking for all n 's divisible by 3.
 $\therefore 1 + 1 + a + b = 3$ or 9 or 12 or 15 or 18
 $\therefore a + b = 1$ or 4 or 7 or 10 or 13 or 16
 $\therefore a + b = 1$ or 7 or 13 is not possible as the sum of two odd numbers cannot be odd.
 $\therefore (a, b) = (1, 3), (3, 1), (1, 9), (3, 7), (5, 5), (7, 3), (9, 1), (7, 9), (9, 7)$
 $\therefore 9$ elements of S are divisible by 3.

Hence, option 1.

